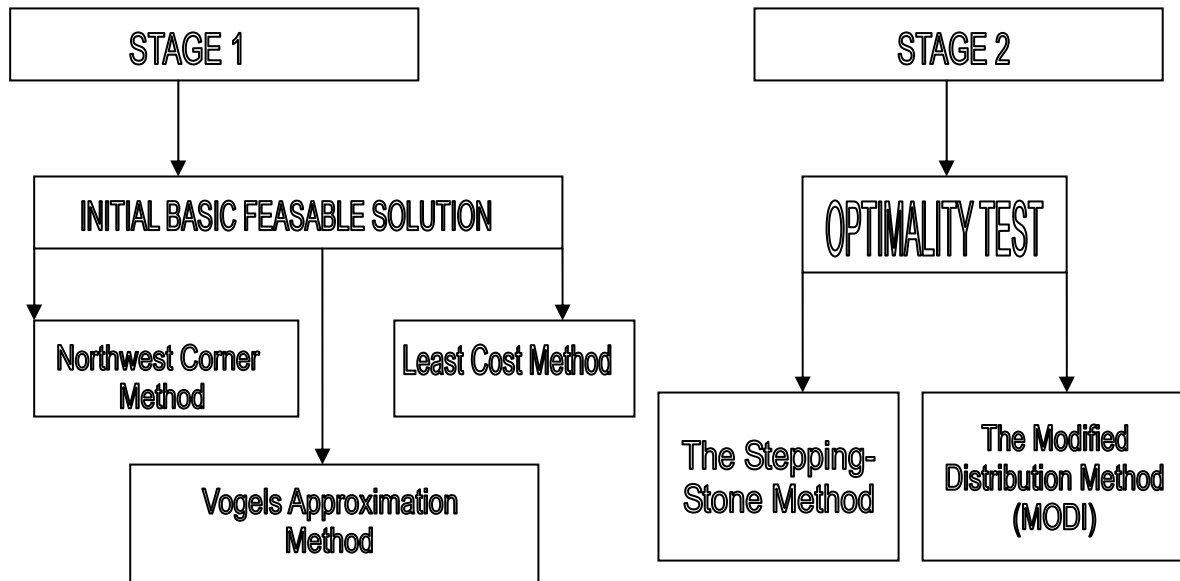


Transportation

Stages in solving transportation problems



Steps for Stage 1 (IBFS)

- 1) If problem is not balanced, introduce dummy row/column to balance it.
[**Note:** Balancing means ensuring that the total of availability and requirement is equal. If total is not equal then we need to make them equal, by introducing dummy row/column.]
- 2) In case of problems of maximization, Convert profit matrix into loss matrix. This is done by subtracting each entry in the table from largest no. in the table.
- 3) Assign M or ∞ to prohibited routes (i.e. routes which are not available) etc.
- 4) Find Initial Basic Feasible Solution using methods defined below.

*[Important Note: 1. Though we can also use step 2 before step 1 (i.e. we can also convert profit matrix in loss matrix before balancing the problem), but, it is recommended to apply the steps in above defined order only.
2. If question is silent, we need to find IBFS using VAM (because it provides least of all values of transportation cost found by different method.)]*

Steps in Northwest Corner Method

1. Allocation always starts from upper left hand corner (i.e. North side first row-first column). Assignment is made in such a way that the resources available are exhausted or demand is fully satisfied.
2. If the resources available are fully exhausted then we move down to second next row and continue the process till the whole demand is exhausted.
3. If the first allocation completely satisfies the demand then we move to the next column of the same row, and continue the allocation process till all availability and requirements are met.
4. The procedure is repeated till all row availability and column requirements are met.

Steps in Least Cost Method

1. Allocation always starts from the cell whose transportation cost per unit is least.
2. The lowest cell is filled as much as possible in view of the availability and destination requirement of its column.
3. Then we move to the next lowest cell and so on continue the procedure in view of the remaining

availability of demand and supply.

4. The procedure is repeated till all row availability and column requirements are met.
5. In case of tie for the lowest cell during allocation, choice may be made for a row or column by which maximum requirement is exhausted.

Steps in Vogel's method

1. Find the difference between two least cost cells in every row and column.
2. Identify the row or column with the highest of the difference. It is in this row or column where allocation should be made.
3. In the row or column selected in step 2, identify the least cost cell. It is in this cell allocation should be made.
4. If there is a tie amongst the largest differences, the choice may be made for a row or column which has least cost. In case there is a tie in cost cell also, choice may be made for a row or column by which maximum requirement is exhausted.
5. Hatch that row or column containing this cell whose totals have been exhausted so that this row or column is ignored in further consideration.
6. Re-compute the row & column differences for the reduced TLP go ahead to Step 3. Continue the above steps till all allocations are made.

Question 1: What are the common methods of obtaining initial feasible solution in a transportation problem?
(3 Marks) Nov./02 & (3 Marks) Nov./08-Old Course

Steps for Stage 2 (Optimality Test)

It can be applied to a transportation table if it satisfies the following conditions:

- a) It contains exactly $m+n-1$ allocations, where m & n represent the no. of rows & columns of the table.

{Note:} (i) In case degeneracy occurs (i.e. allocations are less than $m+n-1$), infinitely small allocation(s) i.e. ϵ is/are introduced in least cost & independent cell to non-degenerate the solution. If least cost cell(s) is/are not independent, then cell(s) with next lower cost(s) is/are chosen.

(ii) Degeneracy can occur at initial basic feasible solution or at a later stage. It may be removed once the purpose is over.}

- b) These allocations are independent i.e. loop can not be performed by them.

{Note:} (i) Closed path or loop should have even no. of turns & is formed with an allocation on each corner, which in turn is a join of horizontal & vertical (not diagonal) lines. A loop may not involve all the allocations.

(ii) Generally, IBFS obtained by Northwest Corner rule or VAM are always in independent positions though they may be $m+n-1$ or less than $m+n-1$ in no.}

The Stepping-Stone Method > **LEAST Important Topic**

1. Start with an arbitrary empty cell (a cell without allocation a.k.a. non-basic variables) and allocated +1 unit to this cell.
2. In order to maintain conditions of requirement & supply (a.k.a. rim constraints), 1 unit will be deducted/added to basic variables (i.e. cells having allocations).
3. Calculate Net change (a.k.a. opportunity cost or net evaluation) in transportation cost as a result of this perturbation.
4. Calculation Net evaluation for every empty cell.
5. If any cell evaluation is -ve, the cost can be reduced so that the solution under consideration can be improved i.e. it is not optimal. On the other hand if all net evaluation are +ve, the given solution is optimal one. If any net evaluation is Zero, the given solution is optimal but there exist other optimal solutions as well.

6. If solution is not optimal, the cell with largest -ve opportunity cost should be selected & allocate maximum units to this route (subject to rim constraints).
7. Iterate towards optimal solution, if further savings are possible.

Modified Distribution Method (MODI) -Preferable > MOST Important Topic

1. Form UV Matrix. It is formed
 - (a) By adding a column to the RHS of transportation table labeled u_i & row in the bottom to it labeled v_j .
 - (b) Assign any value arbitrarily to a row or column variable u_i or v_j .
 - (c) Taking each cost cell $(u_i+v_j)=c_{ij}$, calculate individually all values of u_i & v_j
2. Calculate Net evaluation table, opportunity cost for each unallocated cell i.e. Δ_{ij} i.e. $c_{ij}-(u_i+v_j)$ {Also, Opportunity cost = Actual cost - Implicit cost}
3. Check for signs of Δ_{ij} . If all Δ_{ij} are +ve, the given solution is optimal one. If any Δ_{ij} is Zero, the given solution is optimal but there exist other optimal solutions as well. If there are -ve Δ_{ij} , it means this solution is not optimal.
4. If solution is not optimal, the cell with largest -ve opportunity cost should be selected. Form a closed loop starting and ending at the selected worst -ve cell and reallocate the solution (maintaining rim constraints).

E.g. If Reallocation is to done by transferring the max. amount to the ticked cell, the rule for obtaining max. amount (say $\theta_{\max}$) that can be transferred to the ticked cell is derived as:

Diagram 1: A closed loop with cells containing 3, 1, 9, and 4. The amount to be transferred is θ .

$$\theta_{\max} = \min \text{ of } \begin{cases} 3 - \theta = 0 \\ 9 - \theta = 0 \end{cases} = 3$$

Diagram 2: A closed loop with cells containing 4, 6, 7, and 8. The amount to be transferred is θ .

$$\theta_{\max} = \min \text{ of } \begin{cases} 4 - \theta = 0 \\ 8 - \theta = 0 \\ 9 - \theta = 0 \end{cases} = 4$$

8. Iterate towards optimal solution, if further savings are possible.

Question 2: Obtain the IBFS for the following using (i) North West Corner Rule (ii) LCM (iii) VAM.

	Warehouse-1	Warehouse-2	Warehouse-3	Supply
Factory-1	6	8	4	14
Factory-2	4	9	8	12
Factory-3	1	2	6	5
Demand	6	10	15	-

Question 3: Goods manufactured at 3 plants, A, B and C are required to be transported to sales outlets X, Y and Z. The unit costs of transporting the goods from the plants to the outlets are given below:

Sales outlets\Plants	A	B	C	Total Demand
X	3	9	6	20
Y	4	4	6	40
Z	8	3	5	60
Total supply	40	50	30	120

You are required to:

- (i) Compute the initial allocation by North-West Corner Rule.
(ii) Compute the initial allocation by Vogel's approximation method and check whether it is optional.
(iii) State your analysis on the optionality of allocation under North-West corner Rule and Vogel's Approximation method.

(10 Marks) May/08

[Ans.:

(i) Initial allocation under NW corner rule is:				(ii) Initial allocation under VAM is:			
Plant	Sales Outlet	Units	Initial Cost	Plant	Sales Outlet	Units	Initial Cost
A	X	20	60	A	X	20	60
A	X	20	80	A	X	20	80
B	Y	20	80	B	Y	50	150
B	Y	30	90	C	Y	20	120
C	Z	30	150	C	Z	10	100
			460				460
				It is optimal solution			

(iii) The solution under VAM is optimal with a zero in R_2C_2 which means that the cell C_2R_2 which means that the cell C_2R_2 can come into solution, which will be another optimal solution. Under NWC rule the initial allocation had C_2R_2 and the total cost was the same Rs. 460 as the total cost under optimal VAM solution. Thus, in this problem, both methods have yielded the optimal solution under the 1st allocation. If we do an optimality test for the solution, we will get a zero for Δ_{ij} in C_3R_2 indicating the other optimal solution which was obtained under VAM.]

Question 4 (Minimization-Unbalanced): A product is manufactured by four factories A, B, C & D. The unit production costs are Rs. 2, Rs. 3, Re. 1 and Rs. 5 respectively. Their daily production capacities are 50, 70, 30 and 50 units respectively. These factories supply the product to four stores P, Q, R, S. The demand made by these stores are 25, 35, 105 and 20 units respectively. Unit transportation cost in rupees from each factory to each store is given in the following table:

Factories	Stores			
	P	Q	R	S
A	2	4	6	11
B	10	8	7	5
C	13	3	9	12
D	4	6	8	3

Determine the extent of deliveries from each of the factories to each of the stores so that the total cost (production and transportation together) is minimum.

(10 Marks) May/02

[Ans.: Total Cost Rs. 1465]

Question 5: A company has four terminals U, V, W, and X. At the start of a particular day 10, 4, 6, and 5 trailers respectively are available at these terminals. During the previous night 13, 10, 6, and 6 trailers respectively were loaded at plants A, B, C, and D. The company dispatcher has come up with the costs between the terminals and plants as follows:

	Plants			
	A	B	C	D
U	20	36	10	28
V	40	20	45	20
W	75	35	45	50
X	30	35	40	25

Find the allocation of loaded trailers from plants to terminals in order to minimize transportation cost.

(10 Marks) May/97

[Ans.:

Terminal	Plant	Cost (Rs.)
U	A	80
U	C	60
V	B	60
V	D	20
W	B	210
X	D	125
Transportation Cost		555]

Question 6 (Minimization-balanced-degeneracy): Find optimal solution for the following problem

	Warehouse-1	Warehouse-2	Warehouse-3	Supply
Factory-1	50	30	220	1
Factory-2	30	45	170	3
Factory-3	250	200	50	4
Demand	4	2	2	

[Ans.: Optimal Cost is Rs. 640]

Question 7: The initial allocation of a transportation problem, alongwith the unit cost of transportation from each origin to destination is given below. You are required to arrive at the minimum transportation cost by the Vogel's Approximation method and check for optimality.

(Hint : Candidates may consider $u_1 = 0$ at Row 1 for initial cell evaluation)

					<u>Requirement</u>
11	2 ⁸	8	6 ⁶	2 ⁴	18
9 ¹⁰	9	12	9	6	10
7	6	3 ⁸	7	7	8
9 ²	3	5	6 ²	11	4
<u>Availability</u>					
12	8	8	8	4	40

(6 Marks) May/07

[Ans.: Optimal Cost = Rs. 204]

[Hint for solution: R_4C_2 has the least cost (cost = 3), but this is not independent. The next least cost cell R_4C_3 (cost = 5) is independent, hence we will select it to resolve the degeneracy.]

Question 8 (Minimization-unbalanced-degeneracy): A company has factories at A, B and C which supply warehouses at D, E, F and G. Monthly factory capacities are 160, 150 and 190 units respectively. Monthly warehouse requirements are 80, 90, 110 and 160 units respectively. Unit shipping costs (in rupees) are as follows:

	<u>To</u>			
<u>From</u>	D	E	F	G
A	42	48	38	37
B	40	49	52	51
C	39	38	40	43

Determine the optimum distribution for this company to minimize shipping costs.

[Ans.: Minimal Cost is Rs. 17050]

Question 9: Find optimal solution for the following transportation cost problem.

	Godown-1	Godown-2	Godown-3	Godown-4	Godown-5	Godown-6	Supply
Factory-1	4	6	9	2	7	8	10
Factory-2	3	5	4	8	10	0	12
Factory-3	2	6	9	8	4	13	4
Factory-4	4	4	5	9	3	6	18
Factory-5	9	8	7	3	2	14	20
Demand	8	8	16	3	8	21	-

[Ans.: Rs. 242]

NOTE: SIMILAR QUESTION ↑ IS WRONGLY SOLVED BY STUDY MATERIAL OF ICAI

Question 10 (Prohibited routes): Solve the following transportation problem

	Godown1	Godown2	Godown3	Godown4	Godown5	Godown6	Stock availability
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Factory1	7	5	7	7	5	3	60
Factory2	9	11	6	11	*	5	20
Factory3	11	10	6	2	2	8	90
Factory4	9	10	9	6	9	12	50
Demand	60	20	40	20	40	40	

Note: It is not possible to transport any quantity from factory 2 to godown 5. State whether the solution derived by you is unique. (Nov./1989)

[Ans.: Total Cost Rs. 1120]

Question 11 (Multiple Optimal Solutions): A Compressed Natural Gas (CNG) company has three plants producing gas and four outlets. The cost of transporting gas from different production plants to the outlets, production capacity of each plant & requirement at different outlet is shown in the following cost-matrix table.

Plants	Outlets				Capacity of Production
	A	B	C	D	
X	4	6	8	6	700
Y	3	5	2	5	400
Z	3	9	6	5	600
Requirement	400	450	350	500	1700

Determine a transportation schedule so that the cost is minimized.

The cost in the cost-matrix is given in thousand of rupees.

(10 Marks) Nov./01

[Ans.: Minimum costs Rs. 7350 thousands]

Question 12: A particular product is manufactured in factories A, B, C and D; and is sold at centres 1, 2, and 3. The cost in Rs. of product per unit and capacity in kgms per unit time of each plant is given below:

Factory	Cost (Rs.) per unit	Capacity (kgms) per unit
A	12	100
B	15	20
C	11	60
D	13	80

The sale price in Rs. per unit and the demand in kgms per unit time are as follows:

Sales Centre	Sale Price (Rs.) per unit	Demand (kgms) per unit
1	15	120
2	14	140
3	16	60

Find the optimal sale distribution.

(10 Marks) Nov./97

[Ans.:

From Factory	To Sales Centre	Quantity	Total Profit
A	1	100	300
B	2	20	-20
C	3	60	300
D	1	20	40
D	2	60	60
Dummy	2	60	0
			680]

Question 13 (Investment decision): XYZ and co. has provided the following data seeking your advice on optimum investment strategy:

Beginning of the year	Net Return Data (in paise) of Selected Investments				Amount available (Lacs)
	P	Q	R	S	
1	95	80	70	60	70
2	75	65	60	50	40
3	70	45	50	40	90
4	60	40	40	30	30
Maximum Investment (lacs)	40	50	60	60	-

The following additional information are also provided:

- P, Q, R, and S represent the selected investments.

- The company decided to have four years investment plan.
- The policy of the company is that amount invested in any year will remain so until the end of the fourth year.
- The values (paise) in the table represent net return on investment of one Rupee till the end of the planning horizon (for example, a Rupee invested in Investment P at the beginning of year 1 will grow to Rs. 1.95 by the end of the fourth year, yielding a return of 95 paise).

Using the above, determine the optimum investment strategy.

(10 Marks) Nov/96

[Ans.: Total investment Rs. 13000000]

Question 14: ABC Enterprises is having three plants manufacturing dry-cells, located at different locations. Production cost differs from plant to plant. There are five sales offices of the company located in different regions of the country. The sales prices can differ from region to region. The shipping cost from each plant to each sales office and other data re given by following table:

Production Data Table		
Production cost per unit	Max. capacity in no. of units	Plant no.
20	150	1
22	200	2
18	125	3

Shipping Cost and Demand & Sales Prices table					
Shipping Costs	Sales Office 1	Sales Office 2	Sales Office 3	Sales Office 4	Sales Office 5
Plant 1	1	1	5	9	4
Plant 2	9	7	8	3	6
Plant 3	4	5	3	2	7
Demand & Sales Prices	Sales Office 1	Sales Office 2	Sales Office 3	Sales Office 4	Sales Office 5
Demand	80	100	75	45	125
Sales Price	30	32	31	34	29

Find the production and distribution schedule most profitable to the company.

(10 Marks) Nov./98

[Ans.:

Brand	Set Up	Cost per unit (Rs.)
B ₁	S ₁	4
B ₂	S ₅	5
B ₃	S ₄	6
B ₄	S ₃	7
B ₅	S ₂	5
Total		27

Total Profit Rs. 3170.]

Question 15 (Maximization-unbalanced): A company has three factories and four customers. The company furnishes the following schedule of profit per unit on transportation of its goods to the customers in rupees:

Factory	Customers				Supply
	A	B	C	D	
P	40	25	22	33	100
Q	44	35	30	30	30
R	38	38	28	30	70
Demand	40	20	60	30	

You are required to solve the transportation problem to maximize the profit and determine the resultant of optimal profit.

[Ans.: Maximum Profit Rs. 5130]

Question 16: A leading firm has three auditors. Each auditor can work up to 160 hours during the next month, during which time three projects must be completed. Project 1 will 130 hours, project 2 will take 140 hours, and project 3 will take 160 hours. The amount per hour that can be billed for assigning each auditor to each project is given in Table 1:

Table1

Project

Auditor	1	2	3
	Rs.	Rs.	Rs.
1	1200	1500	1900
2	1400	1300	1200
3	1600	1400	1500

Formulate this as a transportation problem and find the optimal solution. Also find out the maximum total billings during the next month. (10 Marks) May/95

[Ans.:

Auditor	Project	Billing Amount (Rs.)
1	3	304000
2	2	143000
3	1	208000
3	2	42000
	Total billings	<u>697000]</u>

Question 17 (Supply management) The Brown Chemical Company produces a special oil-based material which is currently in short supply. Four of Brown's customers have already placed orders which in total exceed the combined capacity of its two plants and the company needs to know how it should allocate its production capacity to maximize profits.

The following distribution costs per unit have been determined.

Customer	C1 Rs.	C2 Rs.	C3 Rs.	C4 Rs.
Plant X	16	15	14	18
Plant Y	15	15	14	15

The variable unit production costs are Rs. 10 per plant X and Rs. 12 for plant Y. Since the four customers are in different industries, the pricing structure allows different prices to be charged to different customers. (The material undergoes slight variations for each customer at negligible costs). These prices are Rs. 46 for C1, Rs. 42 for C2, Rs. 40 for C3 and Rs. 44 for C4. The customer's orders (in units) are:

C1	C2	C3	C4
2000	5000	3500	2500

and the plant capacities at X and Y in the period concerned are 6000 and 3000 units respectively. Due to an industrial dispute the company can only supply customer C3 from plant Y.

Required:

- (a) Use the transportation algorithm to determine the optimum solution.
 (b) If the industrial disputes were to be resolved so that customer C3 could be supplied from plant X, how would this affect your solution?

[Hint: For easier solution, convert profit matrix in loss matrix before balancing the problem]

[Ans.: For both (a) and (b)

Plants	Customers	Units	Profit (Rs.)
X	C1	1500	30000
X	C2	4500	76500
Y	C1	500	9500
Y	C4	2500	42500
	Total Profit		158500]

Miscellaneous Question:

Question 18 (Overtime Production): A company has factories at A, B, and C, which supply warehouses at D, E, F and G. The factories capacities are 230, 280, and 180 respectively for regular production. If overtime production is utilised, the capacities can be increased to 300, 360 and 190 respectively. Increment unit overtime costs are Rs.5, Rs.4, and Rs.6 respectively. The current warehouse requirements are 165, 175, 205 and 165 respectively. Unit shipping costs in rupees between the factories and the warehouses are

To From	D	E	F	G
A	6	7	8	10
B	4	10	7	6

C	3	22	2	11
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Determine the optimum distribution for the company to minimize costs.

[Ans.:

From	To	Quantity
A	D	30
A	E	175
A	F	25
B	D	115
B	G	165
C	F	180
A ₁	Dummy	70
B ₁	Dummy	60
C ₁	Dummy	10]

Question 19 [VERY IMPORTANT-BECAUSE OF CURRENT ECONOMIC CONDITIONS]: A company has seven manufacturing units situated in different parts of the country. Due to recession it is proposed to close four of these and to concentrate production in the remaining three units. Production in these units will actually be increased from present level and would require an increase in the personnel employed in them. Personnel at the closed units expressed their desire from moving to any one of the remaining units and the company is willing to provide them removal (transfer) costs.

The retraining expenses would have to be incurred as the technology in these units are different. Not all existing personnel can be absorbed by transfer and a number of redundancies will arise, cost of redundancy is given as a general figure at each unit closed.

	A	B	C	D	
No employed:	200	400	300	200	
	(These units A, B, C and D are to be closed)				
Retraining costs in Rs '000/person	A	B	C	D	
Transfer to	E	0.5	0.4	0.6	0.3
	F	0.6	0.4	0.6	0.3
	G	0.5	0.3	0.7	0.3
Removal cost in Rs. '000/person	A	B	C	D	
Transfer to	E	2.5	3.6	3.4	4.7
	F	2.4	4.6	3.4	1.7
	G	2.5	2.7	3.3	2.7
Redundancy payments in	A	B	C	D	
Rs. '000/person :	6.0	5.0	6.0	7.0	
Additional personnel required at					
units :	E	F	G		
Required :	350	450	200		

- Obtain a solution to the problem of the cheapest means to transfer personnel from units closed to those units which will be expanded.
- State with reason whether or not the solution obtained is optimal and unique.
- State the costs of the initial and final solution.

[ICWA /1990]

[Ans.: (i)

From	To	Quantity	Total Cost (Rs. '000s)
A	E	200	600
B	E	100	400
B	G	200	600
B	Dummy	100	500
C	E	50	200
C	F	250	1000
D	F	200	400
			<u>3700</u>

(ii) Since there exist Zero Δ_{ij} hence, solution is optimal but not unique. (iii) Costs are same i.e.Rs. 3700000]

Question 20 (Trans-shipment): A firm has two factories X and Y and three retail stores A, B and C. The number of units of a product available at factories X and Y are 200 and 300 respectively, while demanded at

retail stores are 100, 150 and 250 respectively. Rather than shipping directly from sources to destinations, it is decided to investigate the possibility of trans-shipment. Find the optimal shipping schedule. The transportation costs in rupees per unit are given below.

		Factory		Retail Store		
		X	Y	A	B	C
Factory	X	0	6	7	8	9
	Y	6	0	5	4	3
Retail Store	A	7	2	0	5	1
	B	1	5	1	0	4
	C	8	9	7	6	0

[Ans.: Factory X supplies 100 units each to retail stores A and B. Factory Y supplies 50 units to retail store B and 250 units to C.]